Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

# MEASUREMENT OF GROWTH AND EFFICIENCY IN ARITHMETIC (*Continued*)

S. A. COURTIS
Home and Day School, Detroit

Two of the abilities discussed in the analysis above—the ability to copy correctly, and the ability to "borrow and carry"—are not mentioned in the foregoing tables and graphs. The results of the tests of these abilities were obtained only by analysis of the mistakes made, and of the gross scores. In the case of the first of the two abilities, as the papers were corrected, the number of mistakes in copying made by each grade was noted and reduced, for purposes of comparison, to mistakes per hundred examples. The record is given in Table II. The total number of mistakes per hundred examples is also given, and the percentage one is of the other. Fig. 8 shows the last relation graphically.

TABLE II

MISTAKES IN COPYING

| GRADE | FUNDAMENTALS | | | | | | REASONING | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SEPTEMBER | | | JUNE | | | SEPTEMBER | | | JUNE | | |
| | C* | T† | Percentage | C* | T† | Percentage | C* | T† | Percentage | C* | T† | Percentage |
| 4 | 0.0 | 41 | 0 | 2.8 | 30 | 9 | 8.3 | 80 | 10 | 4.0 | 50 | 8 |
| 5 | 7.7 | 28 | 28 | 4.2 | 38 | 11 | 3.4 | 49 | 7 | 4.5 | 36 | 13 |
| 6 | 7.7 | 26 | 30 | 3.4 | 36 | 9 | 11.0 | 45 | 24 | 3.9 | 48 | 8 |
| 7 | 11.5 | 29 | 40 | 1.8 | 14 | 11 | 7.1 | 42 | 17 | 1.6 | 23 | 7 |
| 8 | 3.9 | 15 | 26 | 5.6 | 21 | 27 | 4.6 | 29 | 16 | 1.9 | 24 | 8 |
| 9 | 8.0 | 30 | 26 | 8.0 | 34 | 24 | 2.7 | 26 | 10 | 4.7 | 26 | 18 |
| 10 | 4.6 | 26 | 18 | 6.1 | 33 | 18 | 3.9 | 27 | 14 | 4.0 | 19 | 21 |
| 11 | 7.2 | 20 | 36 | 4.8 | 39 | 12 | 1.7 | 28 | 6 | 5.7 | 24 | 24 |
| 12 | 6.0 | 24 | 25 | 6.2 | 25 | 25 | 1.4 | 25 | 6 | 4.5 | 21 | 21 |
| 13 | 3.3 | 17 | 19 | 5.8 | 24 | 24 | 2.8 | 25 | 11 | 2.3 | 24 | 10 |

* C—Mistakes in copying.
† T—Total mistakes.

The conclusions to be drawn from the table and figure are many. Chief among these are the facts that the ability in question is a determining factor in successful work, and that it may, or may not, be developed by the *general* work of the classroom. At the beginning of the year a special course was devised for Grade 7 to remedy the marked defect in this ability shown by the test of the previous year and confirmed by the September test under consideration. The success of these efforts is evident from the June results, more perhaps from the actual changes in the number of mistakes given in the table than from the relative values of the graph. On the other hand, Grades 5 and 6 improved during the year without special work, while Grade 8 has for abstract work, at least, suffered an actual loss.

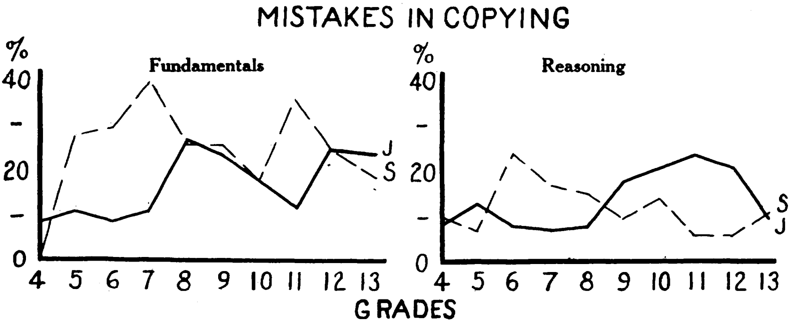

FIG. 8

Nothing is known in regard to the reason for the marked change in Grade 11, nor for the losses in the upper grades in the reasoning test. As these grades do not study arithmetic, any change in ability may be explained as transfer from other school work. Thus, the gain of Grade 11 may possibly be due to transfer from the work in geometry. The ability to pay close attention to the details of any work would seem to be a promising place for transfer to occur, so that the actual losses in one test and the small changes in the other are not easily explained.

In fundamentals, one-fifth of all the mistakes made, and in reasoning one-seventh, was due to pure carelessness in copying. The reasons for the smaller number of such mistakes in the reasoning test are probably (1) the greater attention that is given to the figures in the problems in order to grasp the meaning, and (2) the small proportion the copying is of the total work done. For both tests, however, the results seem to the writer to emphasize the need for cultivating the habit of constant checking. Such a habit would have increased the efficiency in these tests approximately 5 per cent without materially decreasing the amount of work done. Whether or not special exercises and special classroom attention will be generally helpful can be told only by actual trial.

The ability to "borrow and carry" proved more difficult to measure. For an individual finishing eight or more examples in the test on fundamentals, the effects of borrowing and carrying were shown in the larger number of mistakes in the second four examples involving carrying than in the first four in which no carrying was necessary. However, part of the increase was also due to the more difficult combinations involved, and the greater length of the examples. The results by totals are given in Table III.

TABLE III

TOTAL EXAMPLES ATTEMPTED AND RIGHT. PERCENTAGE RIGHT

| | Examples 1 to 4 | | | Examples 5 to 8 | | | Examples 9 to 12 | | |
|---|---|---|---|---|---|---|---|---|---|
| | At. | Rt. | Percentage | At. | Rt. | Percentage | At. | Rt. | Percentage |
| September....... | 943 | 807 | 85 | 837 | 601 | 72 | 347 | 214 | 62 |
| June............ | 903 | 775 | 86 | 834 | 574 | 69 | 498 | 269 | 54 |

DIFFERENCES IN PERCENTAGES OF EXAMPLES RIGHT

| | (Exs. 1–4) – (Exs. 5–8) | (Exs. 1–4) – (Exs. 9–12) | (Exs. 5–8) – (Exs. 9–12) |
|---|---|---|---|
| September.......... | 85–72=13 | 85–62=23 | 72–62=10 |
| June............... | 86–69=17 | 86–54=32 | 69–54=15 |

The results by totals are certainly surprising, to say the least. That 15 per cent of the simple examples without carrying, examples so simple that the answers can easily be obtained mentally, should be worked incorrectly is disappointing from the point of view of the standards of the school. That examples of approximately the same length but involving carrying should be practically double the difficulty of the first is a new idea to the writer. That the increase in the length of Examples 9–12 over that of Examples 5–8 should again increase the difficulty by about the same amount was unsuspected. Finally, that the results in June should be less accurate than in September is simply incomprehensible.

In order to determine whether or not the results shown by the totals were general throughout the grades and in the various operations an analysis of the scores was undertaken. It is evident that in the comparison above, the individuals compared are not the same for all the examples. Nearly all the children finished the first four examples, while but a few were able to reach the twelfth. In the analysis comparison was made, example by example (that is, Examples 1 and 5, 2 and 6, etc.) for the first eight examples, and only the scores of those individuals in each grade were taken who succeeded in finishing both examples of each pair. Table IV gives, for each grade, the number of children that finished each pair of examples, the percentages the examples wrong are of those attempted for each example of the pair, and the difference between these percentages, or the effect of the borrowing and carrying. Fig. 9 gives the last results graphically.

The analysis at least makes plain what is happening through the school although there is little in the results to suggest an explanation. The effect of borrowing and carrying is least in substraction, but the results disclose an effect that was not foreseen. The four negative values in September and the six in June must mean that the procedure in subtraction is so uniform, that borrowing in subtraction becomes so habitual because most of the schoolwork demands it, that the habit persists in those cases in which no borrowing is necessary. That is, most of the classes actually solve the more difficult examples involving carrying more accurately than the simpler ones without it. The fact that the work of the year increased this effect would seem to confirm the explanation given.

For the other operations, the effects of borrowing and carrying in addition are apparently midway between those of subtraction and those of multiplication and division. For the last two operations, the grade variations are so extreme that a judgment of "equally difficult" is perhaps the best. From the average values, the effects in the case of multiplication would appear to be the greater, but from the values given in the table for Grades 9–13, grades which have finished their work in arithmetic, it would seem that the effects in the division are really slightly greater.

TABLE IV

ABILITY TO CARRY

| GRADE | ADDITION EXAMPLES 1 AND 5 | | | | SUBTRACTION EXAMPLES 2 AND 6 | | | | MULTIPLICATION EXAMPLES 3 AND 7 | | | | DIVISION EXAMPLES 4 AND 8 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. Attempted | Percentage Wrong | | Difference | No. Attempted | Percentage Wrong | | Difference | No. Attempted | Percentage Wrong | | Difference | No. Attempted | Percentage Wrong | | Difference |
| September | | Ex. 1 | Ex. 5 | | | Ex. 2 | Ex. 6 | | | Ex. 3 | Ex. 7 | | | Ex. 4 | Ex. 8 | |
| 4 | 12 | 25 | 50 | 25 | .. | .. | .. | .... | .. | .. | .. | .. | .. | .. | .. | .. |
| 5 | 21 | 4 | 30 | 26 | 21 | 14 | 14 | 0 | 10 | 10 | 60 | 50 | .. | .. | .. | .. |
| 6 | 26 | 8 | 16 | 8 | 26 | 8 | 38 | 30 | 25 | 8 | 48 | 40 | 11 | 36 | 64 | 28 |
| 7 | 17 | 6 | 6 | 0 | 17 | 24 | 12 | −12 | 16 | 6 | 50 | 44 | 10 | 30 | 80 | 50 |
| 8 | 30 | 3 | 7 | 4 | 30 | 7 | 7 | 0 | 28 | 0 | 36 | 36 | 22 | 5 | 18 | 13 |
| 9 | 23 | 4 | 40 | 36 | 24 | 29 | 17 | −12 | 24 | 12 | 29 | 17 | 23 | 13 | 39 | 26 |
| 10 | 22 | 14 | 55 | 41 | 22 | 18 | 27 | 9 | 22 | 5 | 18 | 13 | 22 | 5 | 41 | 36 |
| 11 | 27 | 4 | 19 | 16 | 27 | 18 | 11 | − 7 | 27 | 0 | 41 | 41 | 25 | 0 | 24 | 24 |
| 12 | 25 | 16 | 24 | 8 | 25 | 28 | 16 | −12 | 25 | 4 | 20 | 16 | 23 | 9 | 35 | 26 |
| 13 | 24 | 13 | 25 | 12 | 24 | 0 | 4 | 4 | 24 | 0 | 25 | 25 | 23 | 17 | 30 | 13 |
| Average | | | | 18 | | | | 0 | | | | 31 | | | | 27 |
| June | | | | | | | | | | | | | | | | |
| 4 | 10 | 0 | 60 | 60 | 6 | 16 | 0 | −16 | .. | .. | .. | .. | .. | .. | .. | .. |
| 5 | 22 | 9 | 27 | 16 | 22 | 9 | 23 | 14 | 21 | 0 | 48 | 48 | 13 | 54 | 70 | 16 |
| 6 | 23 | 5 | 48 | 43 | 23 | 22 | 9 | −13 | 23 | 0 | 61 | 61 | 19 | 58 | 80 | 22 |
| 7 | 16 | 0 | 6 | 6 | 16 | 0 | 6 | 6 | 16 | 0 | 12 | 12 | 15 | 7 | 40 | 33 |
| 8 | 27 | 4 | 7 | 3 | 27 | 15 | 11 | − 4 | 26 | 4 | 19 | 15 | 25 | 4 | 16 | 12 |
| 9 | 26 | 10 | 35 | 15 | 26 | 19 | 12 | − 7 | 26 | 4 | 50 | 46 | 24 | 25 | 38 | 13 |
| 10 | 20 | 5 | 20 | 15 | 20 | 15 | 15 | 0 | 20 | 0 | 30 | 30 | 20 | 5 | 50 | 45 |
| 11 | 24 | 17 | 33 | 16 | 25 | 12 | 20 | 8 | 25 | 0 | 36 | 36 | 24 | 16 | 54 | 38 |
| 12 | 24 | 8 | 25 | 17 | 24 | 25 | 4 | −21 | 24 | 8 | 37 | 29 | 24 | 8 | 42 | 34 |
| 13 | 24 | 8 | 25 | 17 | 24 | 21 | 8 | −13 | 24 | 8 | 33 | 25 | 24 | 8 | 46 | 38 |
| Average | | | | 21 | | | | − 5 | | | | 34 | | | | 28 |

The inequality of the units of the tests, however, makes anything more than the most general conclusions of questionable value. Perhaps the only safe inference is that the data show the need for a very careful quantitative study of the various

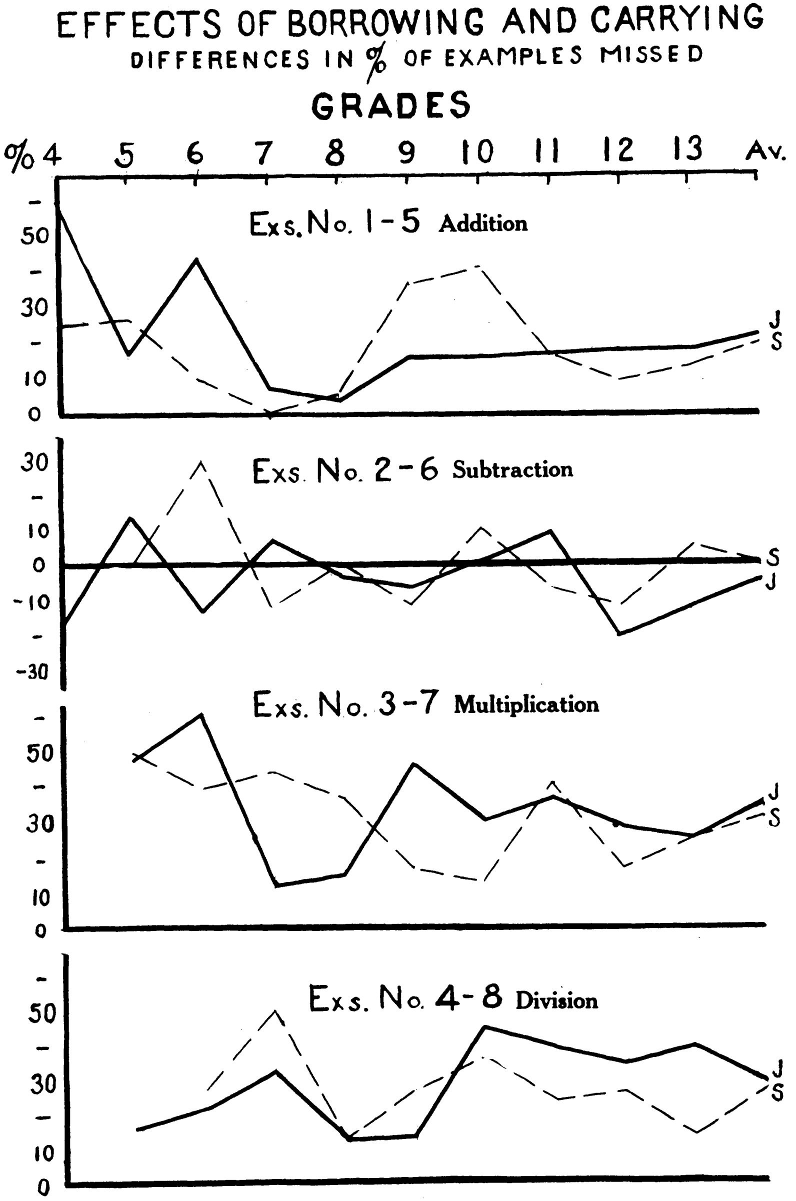

FIG. 9

factors in arithmetical work. If the mere introduction of borrowing and carrying into a multiplication example can cause an increase of 30 per cent in the number of examples missed, the ability under discussion becomes a factor of sufficient importance to receive direct classroom attention.

Perhaps the most striking fact shown by the data is the general increase in the effect of the borrowing and carrying at the end of the year's work. While for the grades actually studying arithmetic (except the erratic sixth) the year's work has, in general, produced some improvement, for most of the high-school grades, it has caused large losses. As the essential elements in borrowing and carrying are memory and attention, it would seem that a year's work in any subject should strengthen rather than weaken this ability and the writer is at a loss to account for the facts shown. In the case of Grade 6, the larger losses are probably due to the marked slump in ability; this grade will later be shown to have suffered near the close of the year (see p. 367) and it may be that something of the same kind took place in all the upper grades.

Although the general tests at the beginning and close of the year measure the total effects of the year's work throughout the school, the results of the special tests, given by the individual teachers in the various classrooms, are of interest as showing for certain grades just how the total effects were produced. The reliability of these results for purposes of comparison from grade to grade is not as great as that of the general tests which were given to the same grades at one time and by one person. However, the results for each grade are consistent with themselves and a comparison of final grade scores (the last week in June) with the previous classroom scores will show practically no difference in the results obtained. At the beginning of the school year, September, 1910, the same tests were again given in the various classrooms, and while at the time this article was commenced, they were not available, they are given here to show the loss during the summer vacation and to complete the story of the year's work. It must be remembered, however, that the last results are not strictly comparable with the others. The various classes change in membership each year, although the losses and gains form but a small fraction of the total enrolment of any grade.

TABLE V

YEAR'S GROWTH IN KNOWLEDGE OF TABLES. GRADE AVERAGES.

| Grade | Op. | Sept. 1909 | Nov. | Dec. | Jan. | Mar. | Mar. | April |  |  |  | May |  |  |  |  | June | Sept. 1910 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  | 4 | 11 | 18 | 25 | 2 | 9 | 16 | 23 | 30 |  |  |
| 5 | + | 35 | 44 | 46 | 47 | 55 | 55 | 63 | 66 | 63 | 63 | .. | 65 | .. | 63 | 65 | 64 | 47 |
|  | − | 30 | 40 | 46 | 45 | 50 | 50 | 48 | .. | 53 | 52 | 52 | 52 | .. | 54 | .. | 54 | 46 |
|  | × | 24 | 32 | 39 | 40 | 45 | 46 | 43 | .. | 52 | .. | 50 | 52 | 52 | 55 | 52 | 56 | 34 |
|  | ÷ | 19 | 30 | 36 | 38 | 41 | 44 | .. | .. | 51 | 50 | 51 | .. | 55 | .. | .. | 50 | 31 |
| 6 | + | 43 | 46 | 45 | 46 | 75 | 65 | 65 | 65 | 65 | 73 | .. | 66 | .. | 63 | 65 | 63 | 56 |
|  | − | 33 | 42 | 43 | 47 | 50 | 51 | 51 | .. | 54 | 53 | 58 | 56 | .. | 55 | 49 | 53 | 53 |
|  | × | 30 | 34 | 49 | 41 | 49 | 50 | 54 | 56 | .. | .. | 58 | 52 | .. | 56 | 56 | 47 | 42 |
|  | ÷ | 26 | 31 | 36 | 36 | 50 | 54 | .. | 50 | .. | 49 | 54 | .. | 56 | 53 | 44 | 48 | 44 |
| 7 | + | 53 | 59 | 68 | 62 | 72 | 79 | 77 | 76 | 82 | 82 | .. | 80 | .. | 83 | 89 | 85 | 79 |
|  | − | 36 | 48 | 54 | 50 | 54 | 60 | 57 | .. | 61 | 64 | 65 | 69 | .. | 69 | 66 | 64 | 59 |
|  | × | 41 | 43 | 48 | 48 | 62 | 59 | 59 | .. | 72 | 70 | 68 | 74 | 72 | 77 | 77 | 74 | 64 |
|  | ÷ | 34 | 43 | 48 | 48 | 59 | 59 | .. | 69 | .. | .. | 67 | .. | 70 | 74 | 74 | 70 | 62 |
| 8 | + | 56 | 65 | 68 | 66 | 73 | 76 | 82 | 83 | 84 | 84 | .. | 82 | .. | 85 | 84 | 84 | 79 |
|  | − | 42 | 52 | 55 | 54 | 59 | 63 | 64 | .. | 67 | 69 | 69 | 70 | .. | 69 | 67 | 68 | 64 |
|  | × | 49 | 55 | 60 | 56 | 62 | 68 | 67 | 68 | .. | .. | 71 | .. | 74 | 73 | 72 | 72 | 60 |
|  | ÷ | 41 | 54 | 58 | 59 | 63 | 66 | .. | 65 | .. | 75 | 74 | 73 | .. | 76 | 70 | 77 | 67 |

From the graph, Fig. 10, it is evident at once that the story of the year's growth varies from grade to grade. Grade 5 shows a rapid gain during the first few weeks of school, a steady uniform growth in all operations during the rest of the year, and a large loss during the summer. Grade 6 shows less gain during the opening weeks, marked fluctuations both in the absolute and relative scores in the various operations during the rest of the year, a marked slump in ability the last six weeks of school, and the smallest loss of any class during the summer. Grade 7 shows the most rapid gain of the four classes, a gain quite uniform throughout. The loss during the summer is small and approximately the same for the various operations. The curve for Grade 8 shows rapid gain during the first weeks, a long "plateau," a marked gain in April, a gain that is held until the close of school, only to be followed by a large loss during the summer.

The cause of these variations is partly known. The writer has strong convictions as to the necessity, place, and amount of drill required in elementary work, and in planning the work for the year it was decided to test out the various methods of drill

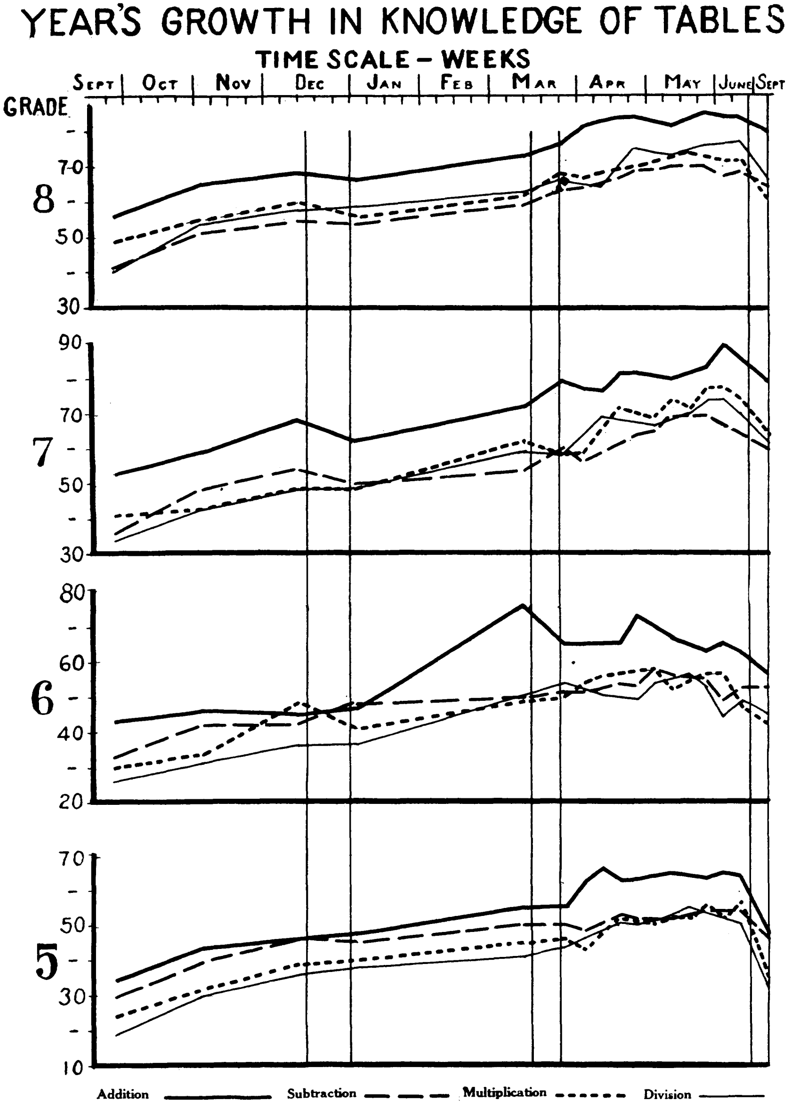

FIG. 10

used at different times in the school. In each of these four grades, the first five minutes of each class period (three per week) is devoted to drill work on the tables. In Grade 5 the spiral method was used, the same table, the same operation, or the same type of practice (visual, motor, oral, etc.) never being followed two days in succession. That is, if the table of 6's in multiplication was written one day, the table of 3's in addition might be practiced orally the next. In Grade 6, however, the drill on one operation was continued many weeks at a time, although the type of practice was frequently changed. In Grade 7 the method followed was a modification of the two extremes of the previous classes, the drill being on one topic for short periods with frequent changes of practice. Grade 8 was taught in three sections, each teacher following a different plan, and the effect as a whole is comparable with that of Grade 5. Unfortunately the press of other duties prevented a careful planning of the experiment and the making of complete records. Unfortunately, also, the observed peculiarities in the behavior of the different grades makes it impossible to say whether the results are due wholly to the methods used, or wholly to the stage of development of the children, or to the effects of one upon the other. The results are, therefore, chiefly valuable as showing the possibilities of such comparative tests.

So far as can be judged, within the limitation mentioned above, continuous drill produces immediate and marked growth followed by equally immediate and marked loss upon cessation of the drill. Note the rise in ability in multiplication in Grade 6 during six weeks in November and December to a position above that of all the other abilities, and the loss of half the gain in the following eighteen days of the Christmas vacation. The curve for addition shows the same effects during the winter term. Compare these ups and downs with the steady, continuous progress of Grade 5. If current opinion were true and "slow growth, sure growth," the method used with Grade 5 would be markedly superior to the others. The loss during the summer vacation, however, opens the whole question anew; for the actual loss that took place may be due to the method, or to the disorganization which, as noted above, makes its appearance regularly at this grade.

The coming year one method of drill will be used with all the grades, and by varying the method from year to year, the best method for each will eventually be determined. It is to be hoped, however, that some superintendent will undertake this same experiment on a larger scale. If the schools of the city were divided into districts of, say, ten schools each, and definite methods of drill assigned to each district, appropriate records of the work actually done being kept, comparative tests like those above would determine in a single year, with scientific exactness, the relative effectiveness of the various methods.

The results of Grade 6 have a bearing, also, upon the question of transfer discussed above. Note that the marked gain in multiplication is accompanied by but slight gain in division, while the curve for substraction is stationary and that for division actually depressed. In the same manner the loss in multiplication has equally little effect on division while the other two operations show recovery and slight gain. This, and other similar effects during the year, suggest some tendency toward equalization within the mind itself. If this proves to be in any degree true, it but serves, in connection with the tendency toward uniformity in relative rank of the various grades in September, to emphasize the importance for education generally of exact quantitative studies of the learning process.